NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: JUNE 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR NEGA CHERE
MODERATOR:	DR DAVID IIYAMBO

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1

Write true if each of the following statements is correct and write false if it is incorrect. Justify your answer.

1.1. If there is a nonzero vector in the kernel of the matrix operator $T_A : \mathbb{R}^n \to \mathbb{R}^n$, then this operator is one to one. [2]

1.2. If the Characteristics polynomial of a 3 - square matrix A is given by $p(\lambda) = \lambda^3 - 4\lambda^2 + 3\lambda - 1$, then tr(A) = -4. [2]

1.3. If λ is a non-zero eigenvalue of an invertible matrix A and v is a corresponding eigenvector, then $1/\lambda$ is an eigenvalue of A^{-1} and v is a corresponding eigenvector. [3]

QUESTION 2

Consider the bases E = $\{e_1, e_2, e_3\} = \{(1,0,\ 0),\ (0,1,\ 0),\ (0,\ 0,\ 1)\}$ and S = $\{u_1, u_2, u_3\}$ = $\{(1, 2,\ 1),\ (2, 5,\ 0),\ (3,\ 3,\ 8\}$ of $\mathbb{R}^3$. Then find the change of basis matrix $P_{E\leftarrow S}$ from S to E.

[6]

QUESTION 3

Let A = $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. Show that $\lambda = -1$ is an eigenvalue of A and find one eigenvector correspondent to this eigenvalue. [12]

QUESTION 4

Let T: $P_2 \to P_2$ defined by $T(a_0 + a_1x + a_2x^2) = a_0 + a_1(x+1) + a_2(x+1)^2$.

4.1. Determine whether T is a linear transformation, if so, find ker (T). [17]

4.2. Determine rank of T and nullity of T and use the result to determine whether T is an isomorphism. [7]

QUESTION 5

Find the coordinate vector $[p(x)]_{\mathcal{B}}$ of $p(x) = 5 + 4x - 3x^2$ with respect to the basis

$\mathcal{B} = \{1 - x,\ 1 + x + x^2,\ 1 - x^2\}$ of P_2. [12]

QUESTION 6

6.1. State the Cayley- Hamilton Theorem. [2]

6.2. Verify the Cayley- Hamilton Theorem for the matrix. [12]

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}.$$

QUESTION 7

Find a 3 x 3 matrix A that has eigenvalues 1, −1, 0 for which $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ are their

corresponding eigenvectors. [16]

QUESTION 8

Find the symmetric matrix that corresponds to the quadratic form

$f(x, y, z) = x^2 + 4xy - 2y^2 + 8xz - 6yz.$ [9]

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER